Coupling of the TE and TM modes of electromagnetic waves in two-dimensional photonic crystals with surface defects of liquid crystals

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We theoretically demonstrate the coupling of the TE and TM modes of electromagnetic waves in two-dimensional photonic crystals with surface defects of liquid crystals. Due to anisotropies of liquid crystals, the TE and TM modes cannot be classified generally in the surface defects, which causes the coupling of the TE and TM modes. The coupling of the TE and TM modes occurs strongly at frequencies at which group velocities of electromagnetic waves become zero, especially at surface defect modes. Possibility of the sharp tunability and the switching of the transmittance in this system has been demonstrated theoretically by the control of directors of liquid crystals by applied electric field due to their anisotropy.

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I. INTRODUCTION

Recently, dielectric periodic structures of optical wavelength order have attracted much attention as photonic crystals from both fundamental and practical viewpoints, because novel concepts such as photonic band gaps have been predicted, and various new applications of the photonic crystals have been proposed [1–3]. In earlier work, two fundamentally new optical principles, that is, the localization of light [4–6] and the controllable inhibition of spontaneous emission of light [7–10] were considered to be the most important. In two-dimensional photonic crystals, the transversal electric (TE) and transversal magnetic (TM) modes of electromagnetic waves can be classified, and properties of photonic crystals in the TE mode differ from those in the TM mode. For example, some photonic crystals possess photonic band gaps only in the TE mode, and other structures of photonic crystals possess photonic band gaps only in the TM mode.

For applications in optical devices, on the other hand, it is important to realize the tunability of photonic crystals, that is, control photonic band structures such as photonic band gaps. Therefore, we have proposed tunable photonic crystals composed of materials whose refractive indices can be changed by outer factors [11,12]. Especially, refractive indices of liquid crystals can be changed by rotating directors of liquid crystals under the influence of electric field. For many applications, it is advantageous to obtain tunable photonic crystals through electro-optic effects. We experimentally proposed to use liquid crystals to photonic crystals and demonstrated tunable photonic crystals with synthetic opals and their replicas infiltrated with liquid crystals [13,14]. Theoretically, Busch and John supported the tunability of photonic crystals infiltrated with liquid crystals [15], and we have also proposed various unique tunable photonic crystals incorporated with liquid crystals [16–18].

In two-dimensional photonic crystals, using liquid crystals, the classification of the TE and TM modes is possible when directors of liquid crystals are parallel and perpendicular to the two-dimensional plane. In such photonic crystals, however, for other director orientations, the TE and TM modes cannot be classified generally due to anisotropies of liquid crystals, that is, the coupling of the TE and TM modes in two-dimensional photonic crystals can occur. Moreover, we can control the mode coupling by rotating directors of liquid crystals under the influence of applied electric field.

In this paper, we theoretically demonstrate the coupling of the TE and TM modes in two-dimensional photonic crystals with surface defects of liquid crystals. The two-dimensional photonic crystals are assumed to be composed of Si circular rods with square lattices. This photonic crystal possesses a photonic band gap only in the TM mode. In the model considered here, surface defect modes appear in the photonic band gap in this mode. We investigate the mode coupling by investigating transmission spectra in the Γ-X direction.

II. THEORY

Circular rods are assumed to be parallel to the z direction. In conventional two-dimensional photonic crystals, magnetic field $H_z(x,y)$ in the TE mode and electric field $E_z(x,y)$ in the TM mode are independent. In two-dimensional photonic crystals with surface defects of liquid crystals, however, $H_z(x,y)$ and $E_z(x,y)$ satisfy the following equation.

$$\begin{bmatrix} L_H(x,y) & L_{HE}(x,y) \\ L_{EH}(x,y) & L_E(x,y) \end{bmatrix}\begin{bmatrix} H_z(x,y) \\ E_z(x,y) \end{bmatrix} + \frac{\omega^2}{c^2}\begin{bmatrix} H_z(x,y) \\ E_z(x,y) \end{bmatrix} = \mathbf{0}, \tag{1}$$

where

$$L_H(x,y) = \frac{\partial}{\partial x}\left[\epsilon_{yy}^{-1}(x,y)\frac{\partial}{\partial x}\right] + \frac{\partial}{\partial y}\left[\epsilon_{xx}^{-1}(x,y)\frac{\partial}{\partial y}\right] - \frac{\partial}{\partial y}\left[\epsilon_{xy}^{-1}(x,y)\frac{\partial}{\partial x}\right] - \frac{\partial}{\partial x}\left[\epsilon_{yx}^{-1}(x,y)\frac{\partial}{\partial y}\right], \tag{2a}$$

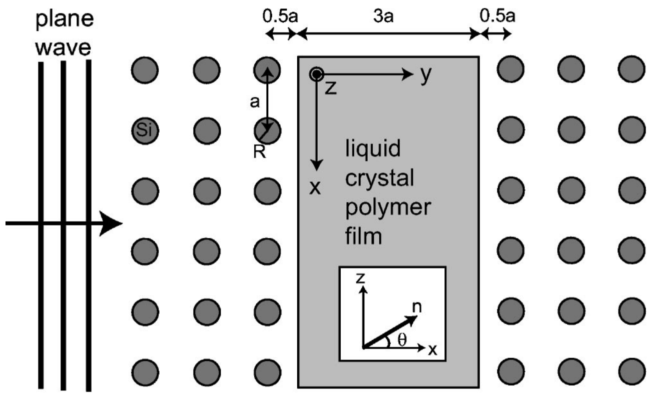

FIG. 1. Schematic model that plane waves are incident on a two-dimensional square-lattice photonic crystal with a surface defect of liquid crystals. R and a indicate radius of the rod and lattice constant of square lattices. The surface defect is assumed to be composed of a liquid crystal polymer film. The arrow **n** and θ in the inset indicate the director and rotation angle of liquid crystals.

$$L_{HE}(x,y)=\frac{i}{\omega\mu_0}\left[\frac{\partial}{\partial y}\left\{\epsilon_{xz}^{-1}(x,y)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right\}-\frac{\partial}{\partial x}\left\{\epsilon_{yz}^{-1}(x,y)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right\}\right], \tag{2b}$$

$$L_{EH}(x,y)=-i\omega\mu_0\left\{\epsilon_{zx}^{-1}(x,y)\frac{\partial}{\partial y}-\epsilon_{zy}^{-1}(x,y)\frac{\partial}{\partial x}\right\}, \tag{2c}$$

$$L_{E}(x,y)=\epsilon_{zz}^{-1}(x,y)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right), \tag{2d}$$

where μ_0 is the permeability of free space and $\epsilon_{ij}(x,y)(i,j=x,y,z)$ is a dielectric tensor.

In the case of director orientations of liquid crystals parallel and perpendicular to two-dimensional planes, $\epsilon_{xz}(x,y)=\epsilon_{zx}(x,y)$ and $\epsilon_{yz}(x,y)=\epsilon_{zy}(x,y)$ are zero and, therefore, the TE and TM modes do not couple, as mentioned earlier.

Figure 1 shows a two-dimensional photonic crystal composed o f circular rods with square lattices with a surface defect of liquid crystals. The surface defect is assumed to be a liquid crystal polymer film, that is, a polymerized liquid crystal. The director **n** of the liquid crystal is expressed by $\mathbf{n}=(\cos\theta,0,\sin\theta)$, where θ is the rotation angle of the director of liquid crystals relative to the x axis in the x-z plane, as shown in the inset of Fig. 1, and then, the dielectric tensor is represented as follows:

$$\epsilon_{xx}(x,y)=\epsilon_o(x,y)\sin^2\theta+\epsilon_e(x,y)\cos^2\theta, \tag{3a}$$

$$\epsilon_{yy}(x,y)=\epsilon_o(x,y), \tag{3b}$$

$$\epsilon_{zz}(x,y)=\epsilon_o(x,y)\cos^2\theta+\epsilon_e(x,y)\sin^2\theta, \tag{3c}$$

$$\epsilon_{xy}(x,y)=\epsilon_{yx}(x,y)=0, \tag{3d}$$

$$\epsilon_{xz}(x,y)=\epsilon_{zx}(x,y)=\{\epsilon_e(x,y)-\epsilon_o(x,y)\}\sin\theta\cos\theta, \tag{3e}$$

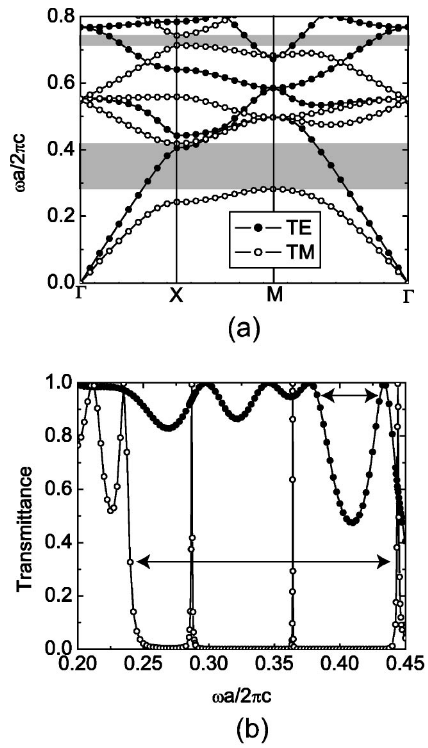

FIG. 2. (a) Photonic band structures of the TE and TM modes in the two-dimensional square-lattice photonic crystal without any defects and (b) transmittances of the TE and TM modes in the Γ-X direction in the two-dimensional square-lattice photonic crystal with the surface defect. Black and white points indicate the TE and TM modes, respectively.

$$\epsilon_{yz}(x,y)=\epsilon_{zy}(x,y)=0, \tag{3f}$$

where $\epsilon_o(x,y)$ and $\epsilon_e(x,y)$ are ordinary and extraordinary dielectric indices, respectively. In the isotropic case, $\epsilon_o(x,y)$ is equal to $\epsilon_e(x,y)$. Since liquid crystals are polymerized, directors of liquid crystals cannot be rotated so much. The changeable angle θ is assumed to be from 0° to 15°. Changes of frequencies of surface defect modes are small, since θ does not change so much. Therefore, we neglect the changes of frequencies of surface defect modes.

Following Sakoda's discussion [19,20], we calculate transmission spectra in the Γ-X direction, as shown in Fig. 1. Flows of electromagnetic waves are defined by Poynting vectors. In the isotropic media, the Poynting vectors of the TE and TM modes are represented as follows:

$$P_{TE}(x,y)=\frac{1}{2}\eta|H_z(x,y)|^2, \tag{4a}$$

$$P_{TM}(x,y)=\frac{1}{2\eta}|E_z(x,y)|^2, \tag{4b}$$

where $\eta=\sqrt{\mu_0/(\epsilon_0\epsilon)}$ is a wave impedance, and ϵ_0 and ϵ are the permittivity of free space and the dielectric index of the isotropic media, respectively. When Poynting vectors of input and output waves are P^{in} and P^{out}, respectively, the transmittance is P^{out}/P^{in}.

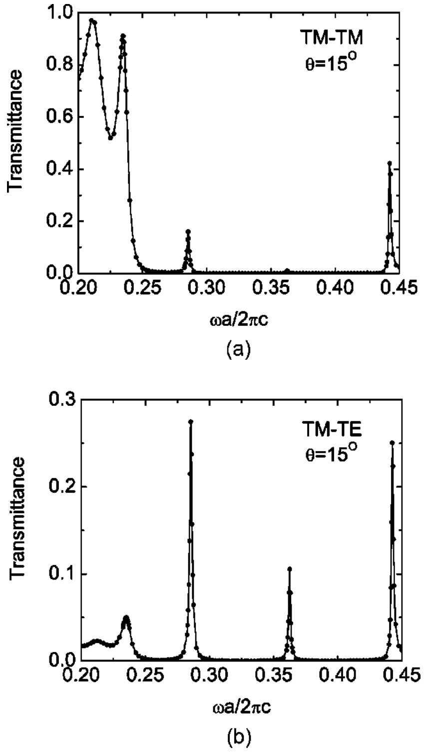

FIG. 3. Transmittances of (a) the output TM wave and (b) the output TE wave when the TM wave is incident on the two-dimensional photonic crystal at $\theta = 15°$.

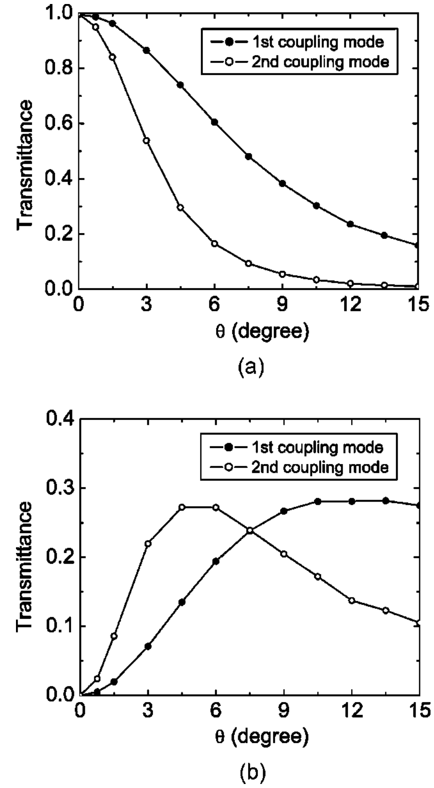

FIG. 4. Dependences of transmittances of (a) the output TM wave and (b) the output TE wave on θ ranging from 0° to 15° when the TM wave is incident on the two-dimensional photonic crystal. Black and white points indicate the first and the second coupling modes, respectively.

III. NUMERICAL CALCULATION AND DISCUSSION

In Fig. 1, circular rods are assumed to be composed of Si. Background is the air. The dielectric index and the radius of a circular rod are $\epsilon = 11.9$ and $R/a = 0.2$. a is the lattice constant of square lattices. Ordinary and extraordinary refractive indices of liquid crystals are $n_o = 1.51$ and $n_e = 1.67$ [CS-1029(CHISSO)], respectively. The width of a surface defect is $3a$, and the interval between the surface defect and a neighboring rod is $0.5a$. Plane waves are incident on the two-dimensional square-lattice photonic crystal with the surface defect, as shown in Fig. 1.

First, we investigate the case where the TE and TM modes do not couple. Figure 2(a) shows photonic band structures of the TE and TM modes in the two-dimensional square-lattice photonic crystal without any defects. Black and white points indicate the TE and TM modes, respectively, and shaded regions indicate photonic band gaps. As shown in Fig. 2(a), photonic band gaps exist only in the TM mode. In Fig. 2(b), we show transmittances in the Γ-X direction in the TE and TM modes at $\theta = 0$ in the two-dimensional square-lattice photonic crystal with the surface defect, as shown in Fig. 1. Black and white points indicate transmittances in the TE and TM modes, respectively. Arrows indicate photonic band gaps in the Γ-X direction. In the TM mode, there exists a wide photonic band gap in the Γ-X direction, and two surface defect modes appear in the photonic band gap. The peak at the higher photonic band edge shifts to the photonic band gap due to the surface defect and, therefore, we consider this peak around $\omega a/2\pi c = 0.45$ as the photonic band edge, since it is not a clear surface defect mode in comparison with the two surface defect modes in the photonic band gap.

In the TE mode, on the other hand, the transmittance decreases in the frequency range of the photonic band gap. However, the transmittance is not so low because of the narrow photonic band gap, as shown in Fig. 2(a).

Next, we investigate transmittances at $\theta = 15°$ when the TM wave is incident on the two-dimensional photonic crystal. Since the coupling of the TE and TM modes occurs at the surface defect, the TE wave is also obtained as the output wave other than the TM wave. When Poynting vectors of the input TM wave, the output TM wave, and the output TE wave are P_{TM}^{in}, P_{TM}^{out}, and P_{TE}^{out}, respectively, transmittances are represented as follows:

$$T_{TM\text{-}TM} = \frac{P_{TM}^{out}}{P_{TM}^{in}}, \tag{5a}$$

$$T_{TM\text{-}TE} = \frac{P_{TE}^{out}}{P_{TM}^{in}}, \tag{5b}$$

where $T_{TM\text{-}TM}$ is the transmittance of the output TM wave and $T_{TM\text{-}TE}$ is the transmittance of the output TE wave. Figures 3(a) and 3(b) show the transmittances $T_{TM\text{-}TM}$ and $T_{TM\text{-}TE}$ at $\theta = 15°$. As shown in Fig. 3(a), the transmittance

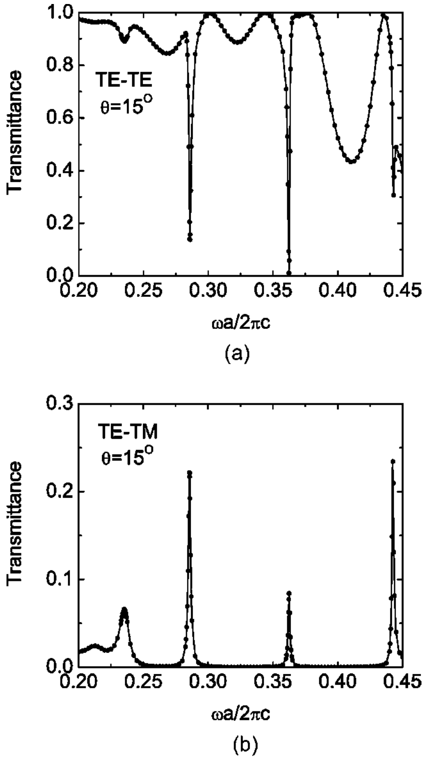

FIG. 5. Transmittances of (a) the output TE wave and (b) the output TM wave when the TE wave is incident on the two-dimensional photonic crystal at $\theta = 15°$.

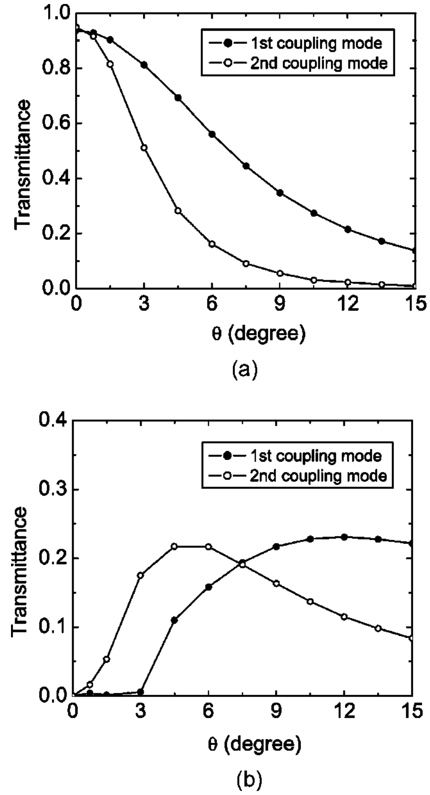

FIG. 6. Dependences of transmittances of (a) the output TE wave and (b) the output TM wave on θ ranging from 0° to 15° when the TE wave is incident on the two-dimensional photonic crystal. Black and white points indicate the first and the second coupling modes, respectively.

$T_{TM\text{-}TM}$ becomes much lower at the two surface defect modes in comparison with the TM mode in Fig. 2(b). As shown in Fig. 3(b), on the other hand, four clear peaks are obtained in the transmittance $T_{TM\text{-}TE}$. These peaks are obtained at frequencies of omnidirectional TM band gap edges and surface defect modes in the TM mode. That is, mode couplings occur strongly when group velocities of electromagnetic waves become zero. Especially, $T_{TM\text{-}TM}$ greatly changes at frequencies of the two surface defect modes. Therefore, we focus our attention on $T_{TM\text{-}TM}$ and $T_{TM\text{-}TE}$ at the frequencies of the two surface defect modes, and define mode couplings at lower and higher frequencies of the two surface defect modes as the first and the second coupling modes, respectively. At the second coupling mode, $T_{TM\text{-}TM}$ mostly becomes zero and, therefore, only the TE wave is obtained as the output wave, which means the conversion to the TE mode from the TM mode although the transmission power decreases.

Figures 4(a) and 4(b) show dependences of $T_{TM\text{-}TM}$ and $T_{TM\text{-}TE}$ on θ ranging from 0° to 15° at the first and the second coupling modes, respectively. As shown in Fig. 4(a), $T_{TM\text{-}TM}$ decreases monotonically with increasing θ, regardless of the first and the second coupling modes. As shown in Fig. 4(b), on the other hand, $T_{TM\text{-}TE}$ exhibits complex behaviors. That is, $T_{TM\text{-}TE}$ at the first coupling mode increases monotonically with increasing θ and becomes maximum, and $T_{TM\text{-}TE}$ at the second coupling mode increases and decreases monotonically after becoming maximum with increasing θ.

Moreover, we investigate transmittances at $\theta = 15°$ when the TE wave is incident on the two-dimensional photonic crystal. Since the coupling of the TE and TM modes occurs at the surface defect, the TM wave is also obtained as the output wave other than the TE wave. When Poynting vectors of the input TE wave, the output TE wave, and the output TM wave are P_{TE}^{in}, P_{TE}^{out}, and P_{TM}^{out}, respectively, transmittances are represented as follows.

$$T_{TE\text{-}TE} = \frac{P_{TE}^{out}}{P_{TE}^{in}}, \tag{6a}$$

$$T_{TE\text{-}TM} = \frac{P_{TM}^{out}}{P_{TE}^{in}}, \tag{6b}$$

where $T_{TE\text{-}TE}$ is the transmittance of the output TE wave and $T_{TE\text{-}TM}$ is the transmittance of the output TM wave. Figures 5(a) and 5(b) show transmittances $T_{TE\text{-}TE}$ and $T_{TE\text{-}TM}$ at $\theta = 15°$. In the TE mode at $\theta = 0°$, transmittances are high at frequencies other than the photonic band gap, as shown in Fig. 2(b). As evident in Fig. 5(a), however, transmittances decrease greatly at the two surface defect modes of the TM mode in Fig. 2(b). As shown in Fig. 5(b), on the other hand, four clear peaks are obtained in the transmittance. These peaks are obtained at frequencies of omnidirectional TM band gap edges and surface defect modes in the TM mode. That is, the mode couplings occur strongly when group velocities of electromagnetic waves become zero. Especially, $T_{TE\text{-}TE}$ greatly changes at frequencies of the two surface defect modes. Therefore, we focus our attention on $T_{TE\text{-}TE}$ and $T_{TE\text{-}TM}$ at the frequencies of the two surface defect modes, and define mode couplings at lower and higher frequencies of the two surface defect modes as the first and the second coupling modes, respectively. At the second coupling mode, $T_{TE\text{-}TE}$ mostly becomes zero and, therefore, only the TM wave is obtained as the output wave, which means the conversion to the TM mode from the TE mode although the transmission power decreases.

Figures 6(a) and 6(b) show dependences of $T_{TE\text{-}TE}$ and $T_{TE\text{-}TM}$ on θ ranging from 0° to 15° at the first and the second coupling modes, respectively. As shown in Fig. 6(a), $T_{TE\text{-}TE}$ decreases monotonically with increasing θ, regardless of the first and the second coupling modes. As shown in Fig. 6(b), on the other hand, $T_{TE\text{-}TM}$ exhibits complex behaviors. That is, $T_{TE\text{-}TM}$ at the first coupling mode begins to increase around $\theta = 3°$ and increases monotonically with increasing θ and becomes maximum. However, $T_{TE\text{-}TM}$ at the second coupling mode increases and decreases monotonically after becoming maximum with increasing θ.

In two-dimensional photonic crystals with surface defects of liquid crystals, mode couplings occur strongly when group velocities of electromagnetic waves become zero. Especially, original transmittances of the TE and TM modes change greatly at surface defect modes of the TM mode when directors of liquid crystals are not parallel or perpendicular to two-dimensional planes. Therefore, we can obtain the sharp tunability that transmittances change only at certain frequencies by rotating directors of liquid crystals under the influence of applied electric field.

IV. CONCLUSION

In conclusion, we theoretically demonstrated the coupling of the TE and TM modes in two-dimensional photonic crystals with surface defects of liquid crystals. Transmittances of the output TM and TE waves are calculated numerically when the input TM wave is incident on the two-dimensional photonic crystal, and transmittances of the output TE and TM waves are also calculated numerically when the input TE wave is incident. Regardless of the input TE and TM waves, the coupling of the TE and TM modes occur strongly at frequencies at which group velocities of electromagnetic waves become zero, especially at the two surface defect modes. At a higher frequency of the two surface defect modes, the conversions of polarized vectors of electromagnetic waves can be obtained although transmission powers decrease. This photonic crystal may provide the sharp tunability that transmittances change only at certain frequencies by rotating directors of liquid crystals under the influence of electric field.

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